

# Evaluation of Limit-State-Functions by means of a Diffusion Model in the Risk Assessment of a Biodegradable Pollutant Discharge in a River Pathway

G. Maria\* and C. Maria\*\*

\* Laboratory of Chemical & Biochemical Reaction Engineering, University Politehnica of Bucharest, P.O. 35-107 Bucharest,  
Phone: +40 744 830308, Email: gmaria99m@hotmail.com, WWW: <http://www.cael.pub.ro/echipa/webMG/INDEX/index.html>

\*\* Research & Engineering Institute for Environment (ICIM), Spl. Independentei 294, Bucharest  
E-Mail: c\_maria100@hotmail.com, WWW: <http://www.icim.ro/index.html>

**Abstract:** When evaluate consequences of a wastewater treatment (WWT) system failure, by using a probabilistic analytical method of risk assessment, derivation of the so-called limit state functions (LSF) are necessary. For the case study of an accidental pollutant discharge from a municipal WWT-plant, the LSF have been accurately evaluated by simulating the contaminant fate and transport in a control section of the riverine pathway by using a diffusion model with accounting for pollutant biodegradation.

**Keywords:** risk assessment, pollutant riverine dispersion

## 1. Introduction

Municipal wastewaters contain a large variety of contaminants (organic, inorganic, micro-organisms, suspended solids), coming from a variety of sources and presenting important fluctuations both in flow-rates and composition. A classical WWT plant consists in a series of sections: primary (mechanical and chemical), secondary (biological) and, in modern configurations, a tertiary (advanced) pollutant treatment. The biological treatment is the most complex step in removing organics (BOD) and inorganic pollutants from wastewaters, being conducted on an acclimatized activated sludge in coupled aeration basin – sludge settler units. This step is very sensitive to input-flow oscillations, operating conditions and biomass evolution. Sudden increases in substrate concentration, some inhibitory substances, deterioration of the biomass, or the low flexibility of the aerator-settler unit, all these can lead to a difficult process identification, control and optimization (see for instance reviews of Maria et al. [1-3], Tchobanoglous & Burton [4], and Gray [5]). Several biological WWT improvements have been reported [6]: (i) the use of sequential environments / WWT-units for enhanced bio-transformation, by accumulating the desired micro-organisms via operation modes, recommended nutrients, additives and sources of organics; (ii) WWT process flow-sheet optimization by including serial-parallel aerobic, anaerobic, and anoxic cycles, with multiple recycling loops; (iii) integration of chemical and biological processes for inducing an increased bio-availability by means of: a preliminary chemical oxidation of recalcitrant compounds; a chemical 'polish' to avoid low-recalcitrant compounds; a chemical pre-oxidation of inhibitory compounds; a chemical post-treatment of products followed by further biodegradation; improvement of the

WWT-bioreactor performance (e.g. the use of high-rate biofilms and membranes).

In spite of the mentioned modern solutions, the WWT-plant safe operation can become a critical issue, being related to the maximum input loads that can be safely processed. As a consequence, pollutant loads in the WWT-outputs exceeding regulation standards can accidental occur, and the risk management must consider the probability of the WWT-plant failure. To assess the risk of an accidental pollutant release in a river, simulation of the discharge scenario over a river control section combined with a probabilistic analysis are necessary. As random variables ( $\mathbf{u}$ ) on which the risk depends, pollutant flow-rates or loads in the WWT-effluent, or various WWT operating parameters can be considered. If normal variables are assumed, the mean ( $\mu_{u_i}$ ) and variance ( $\sigma_{u_i}^2$ ) are determined from the WWT-plant operation records and accident statistics (if any).

In order to perform a complete WWT-plant risk assessment, the present study aims to exemplify the methodology to construct the so-called LSF-functions associated with the violation of polluting constraints over a certain river control area, at locations downstream the release point. The risk is here defined as the probability that a given location hazard exceeds a set of defined limits. The discharge scenario, referring to a municipal WWT-plant failure, is simulated by means of a relatively simple diffusion model with accounting for the pollutant biodegradability. Subsequent use of random variables with known characteristics can lead to LSF-functions in probabilistic terms, and to evaluate the failure probability ( $p_f$ ) over the river control section.

## 2. Risk assessment measures

Risk assessments are already routine methods to evaluate the failure probability of an engineering system. The risk, defined as the product between the failure probability ( $p_f$ ) and the consequences of a future event, can be evaluated by means of sampling or analytical probabilistic methods (see for instance the reviews of Chamis et al. [7], Su [8], Wu et al. [9], Anghel [10], and the large number of software-packages, such as VeroSolve [11] or Crystal Ball described by Anghel [10]). Generally, the 'risk' is a quantifiable measure of the safety of a system and, because it refers to a future (possible) event, it is subjected to uncertainties being defined in probabilistic terms.

In the analytical methods, if one denotes with  $\mathbf{u}$  a  $n$ -dimensional random variable (or parameter-vector) on which the system performance and risk depend, a set of functions  $\mathbf{g}(\mathbf{u})$  can be defined, i.e. the so-called LSF, such that violation of constraints of type  $\mathbf{g}(\mathbf{u}) < \theta$  to be assimilated with the system failure. For multiple defect sources, the system reliability method must simultaneously account for  $n$ -failure events  $F_1, \dots, F_n$ , related to the  $\mathbf{g}$ -functions and  $n$ -random variables ( $\mathbf{u}$ ) causing the defects. The probability of system failure is then expressed as a union of the failure events, that is:

$$p_f = P[F_1 \cup F_2 \cup \dots \cup F_n] = P\{\mathbf{g}(\mathbf{u}) < \theta\} = \int_{\mathbf{g}(\mathbf{u}) < \theta} \dots \int f(\mathbf{u}) d\mathbf{u}. \quad (1)$$

(where  $p_f$  = failure probability;  $P$  = probability;  $f(\mathbf{x})$  = joint probability density function of  $\mathbf{u}$ ;  $\mathbf{g}$  = LSF constraint functions;  $\mathbf{u}$  =  $n$ -dimensional random variable). For independent or weakly correlated events, an approximate formula for  $p_f$  is [9]:  $p_f = \sum_i p_i$  (where  $p_i$  = the individual failure occurrence probability).

One approximate route to evaluate  $p_f$  is the first-order second moment method developed by Hasofer & Lind (see review of Su [8]), i.e. the so-called 'Most Probable Failure Point' (MPP) method. MPP is based on a *safety index*  $\beta > 0$ , which is defined as the shortest distance between the origin of the reduced coordinate system (in terms of  $u'_i$ ) and the *failure surface* defined for every constraint by  $LSF(\mathbf{u}) = \mathbf{g}(\mathbf{u}) = 0$ , i.e. [8]:

$$p_f = \Phi(-\beta(\mathbf{u}')); \quad u'_i = \frac{u_i - \mu_{u_i}}{\sigma_{u_i}}; \quad u'_i = -\alpha_i^* \beta; \\ \alpha_i^* = (\partial g / \partial u'_i) / \sqrt{\sum_i (\partial g / \partial u'_i)^2}; \quad i = 1, \dots, n. \quad (2)$$

(where  $\mu_{u_i}$  = mean of the variable  $u_i$ ;  $\sigma_{u_i}$  = standard deviation of  $u_i$ ;  $\alpha_i^*$  = direction cosines;  $\Phi$  = the cumulative distribution of the standard normal variate). In the MPP method, variables are assumed to be normally

distributed, with known mean and variance. Safety index  $\beta$  is evaluated by (numerically) solving the equation:

$$g(u_1, \dots, u_n) = 0; \quad u_i = \mu_{u_i} - \alpha_i^* \sigma_{u_i} \beta; \\ (\partial g / \partial u'_i) = (\partial g / \partial u_i) \sigma_{u_i}; \quad i = 1, \dots, n. \quad (3)$$

LSF depends on the process characteristics (performance), on the random variable distribution, but also on the set of admissible constraints (physico-chemical, technological, safety). In the analytical variant, LSF functions are generated by means of the process model. A convenient way is to consider, in a first step, the deterministic process and to simulate the system failure based on the averages  $\mu_{\mathbf{u}}$ . Then, by replacing the random variables in the model, i.e.  $u_i = \mu_{u_i} - \alpha_i^* \sigma_{u_i} \beta$ , a stochastic solution for LSF and a safety index can be generated.

## 3. Modelling pollutant fate and dispersion in the riverine pathway

In order to simulate a hypothetical river contamination with an accidental discharge from a WWT-plant, a stationary diffusion model has been adopted [12]:

$$w \frac{\partial C}{\partial x} = D_y \frac{\partial^2 C}{\partial y^2} - r; \\ \frac{\partial C}{\partial y} \Big|_{y=0}^{y=B} = 0; \quad C|_{y=B} = 0; \quad C|_{x=0}^{y=0} = C_o; \\ C_o = C_{fond} + Q_{poll} / Q_{river}; \\ Q_{river} + Q_{eff} = w \cdot 2B \cdot h, \quad (4)$$

(where  $C_{fond}$ ,  $C$  = pollutant concentration in the river before and at contamination source;  $x$  = longitudinal distance from source downstream the river;  $y$  = lateral distance from middle-river;  $B$  = river half-width;  $Q_{river}$ ,  $Q_{eff}$ ,  $Q_{poll}$  = river, effluent, pollutant flow-rates respectively;  $w$  = water-flow average velocity;  $h$  = river depth;  $D_y$  = apparent radial dispersion coefficient;  $r$  = pollutant biodegradation rate). Such a model is based on several simplificatory assumptions: i) a small size discharge source located in the middle of the river; ii) a uniform flow with constant flow-rates; iii) a quasi-uniform river-size over the control area (of rectangular cross-section); iv) a constant biodegradation rate in the analysed site; v) a negligible contaminant adsorption / desorption from river particles or sediments; vi) a contaminant release longer than the travel-time in the control section (i.e. steady-state solution); vii) an advection which dominates dispersion in the longitudinal direction; viii) a fully mixed contaminant plume over the river depth; ix) a dispersion coefficient ( $D_y$ ) that includes the lateral turbulent mixing and diffusion (adopted value of  $D_y = 0.06hw$ , [13]). Fore

more detailed pollutant dispersion models, the reader is referred to the literature (see for instance the reviews of Roman [14], and Whelan & McDonald [12]).

If a first-order pollutant biodegradation kinetics is assumed ( $r = kC$ ), an analytical solution of model (4) is possible to be derived, of the form:

$$C(x, y, t_c) = (C_{fond} + C_{disp}(x, y)) \times \exp(-kt_c), \quad (5)$$

(where  $C_{disp}$  = dispersed pollutant concentration at various locations downstream the river;  $t_c = x/w =$  pollutant residence time from the source to the  $x$ -distance). If a more complex pollutant biodegradation kinetics is considered [15], a numerical solution of model (4) can be obtained by using the finite difference methods [16].

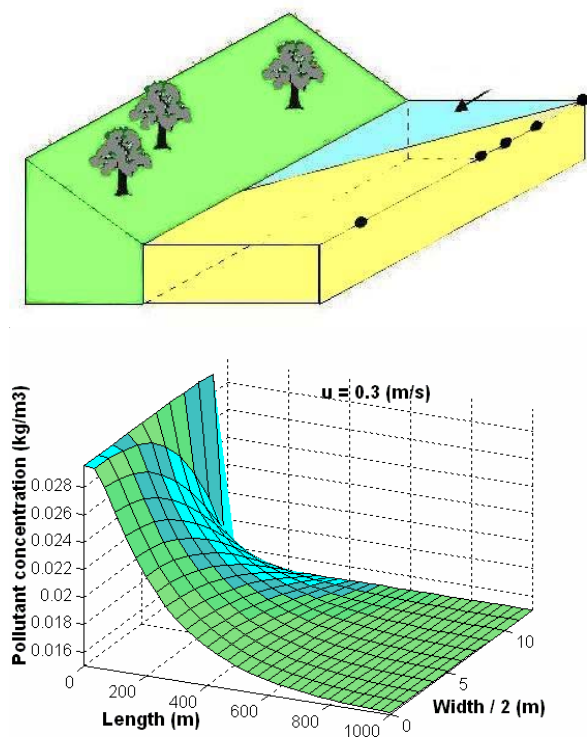


Figure 1. Riverside contamination pathway (Up-figure). Pollutant 3D distribution over a control section (Down-figure).

#### 4. Deriving the LSF and risk assessment - A case study

To simulate a municipal WWT-plant failure and an accidental pollutant dispersion in a river, downstream the plant location, simple LSF constraint functions have been defined for the BOD-organics target pollutant:

$$LSF(\mathbf{u}) = g(\mathbf{u}) = C_{adm} - C(x, y, t_c, \mathbf{u}), \quad (6)$$

(where  $\mathbf{u}$  = random independent variables causing the risk, such as pollutant concentration at source,  $C_o$ ). The considered numerical values are the followings (see notations below eq. 4):  $0 < x < 1$  km;  $0 < y < B = 12.5$  m;  $h = 5$  m;  $Q_{river} = 35$  m³/s;  $Q_{poll} = 0.55$  kg/s;  $Q_{eff} = 2.5$  m³/s;  $C_{fond} = 0.015$  kg/m³;  $r = kC$ ;  $k = 0.05$  day<sup>-1</sup> [17];  $C_{adm} = 0.015$  kg/m³ [18].

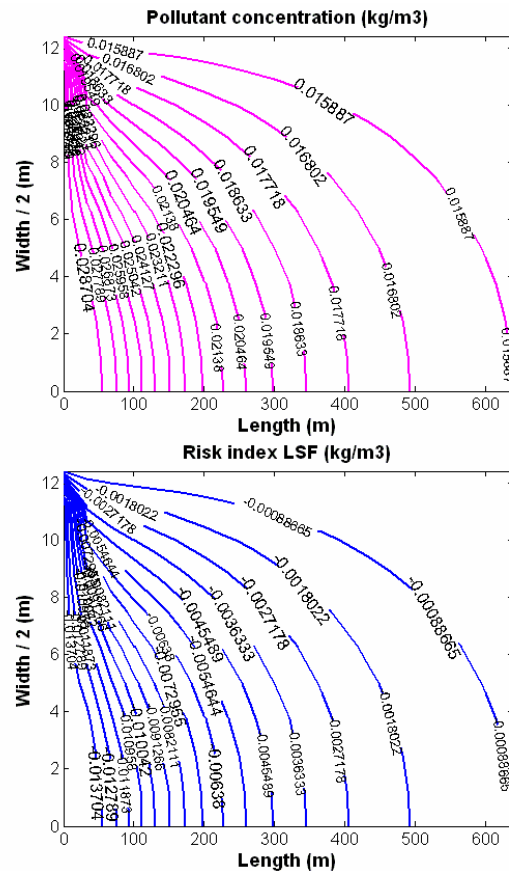


Figure 2. Pollutant distribution over the river pathway (iso-concentration curves; Up-figure). LSF risk index in the control section (Down-figure).

Simulation of the pollutant fate and transport in the river control pathway are presented in Figure 1 for an average  $\mu_u = C_o$ , while the obtained failure region of LSF  $< 0$  is plotted in Figure 2. It is to observe that the risk area, presenting significant negative LSF values, is of ca. 1 km downstream the discharge location; the most affected is the middle-river area comparatively to the bank proximity.

In the next analysis step (not presented here), specifications of the random variables  $\mathbf{u}$  (i.e.  $\mu_{u_i}, \sigma_{u_i}$ ) for the considered WWT-plant are substituted in the LSF definition (6), leading to determine the safety index  $\beta$  by means of eq. (3) and of the failure probability  $0 \leq p_f \leq 1$  by means of eq. (2).

## 5. Conclusions

The presented diffusion model to simulate the pollutant dispersion in a riverine pathway, downstream from a contamination source, can be successful used to simulate an accidental WWT-plant failure scenario. The derived risk LSF-measures together with a complete risk assessment based on a probabilistic analysis can be used to base failure prevention analyses, plant optimization, risk management and river-pollution monitoring measures, and an environmental impact evaluation.

The probabilistic risk analysis depends on the used dispersion model quality, biodegradation kinetics adequacy, WWT-plant effluent random characteristics, and on the river flowing regime.

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